Shims with Curvature

Paul Jung

May 30, 2017

To map the shims with curvature we would effectively like to map the coordinates: $x \to r$ and $y \to \theta$. So that is accomplished with the transformation:

$$w = e^z$$

where $z = x + iy$ so it could be expressed as:

$$w = e^{x+iy} = e^x e^{iy} = e^x(\cos(y) + i\sin(y))$$

So the new coordinates would be:

$$x' = \Re(w) = e^x \cos(y)$$

$$y' = \Im(w) = e^x \sin(y)$$

The domains of interest are shown in fig. 1, the square domain

$$\{(x, y) : x \in [a, b], y \in [-\pi/3, \pi/3]\},$$

gets mapped to fig. 2 the curved domain:

$$\{(r, \theta) : r \in [a, b], \theta \in [-\pi/3, \pi/3]\}$$

Note the entire y-axis gets mapped to the origin, with a non-unique representation.

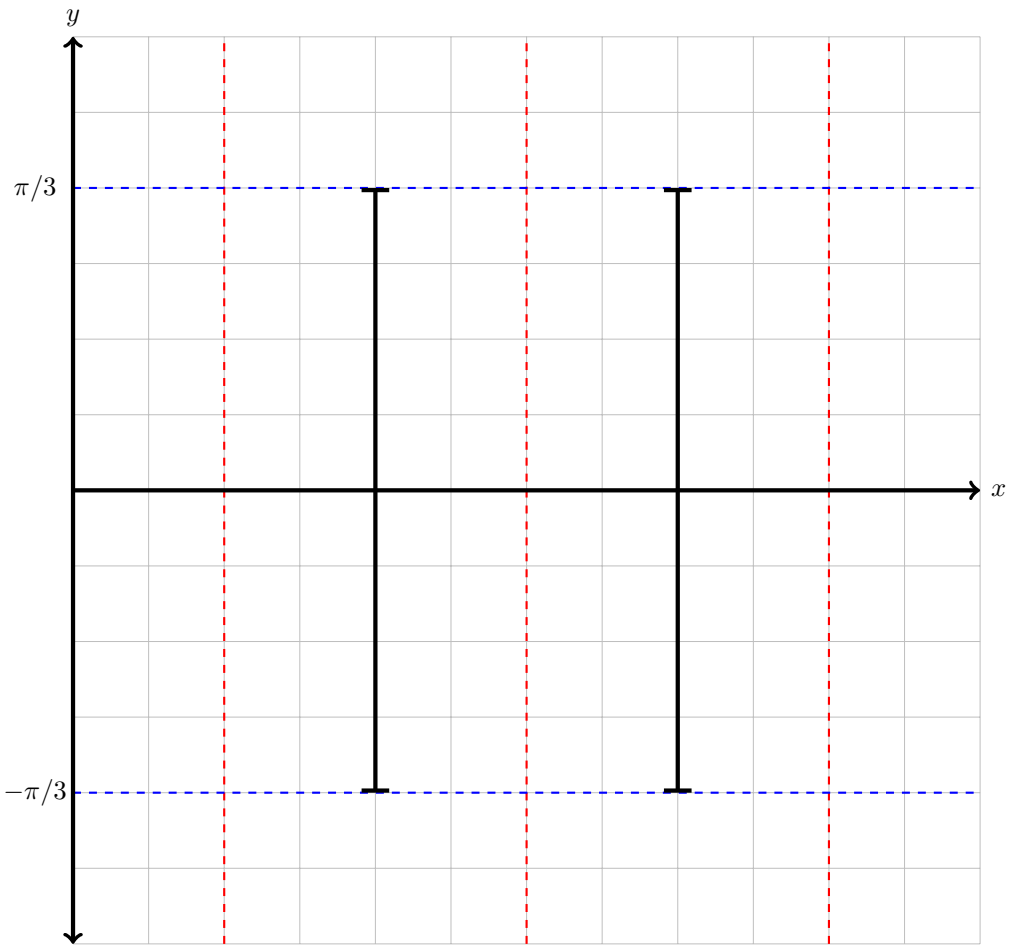

Figure 1: Straight Shims

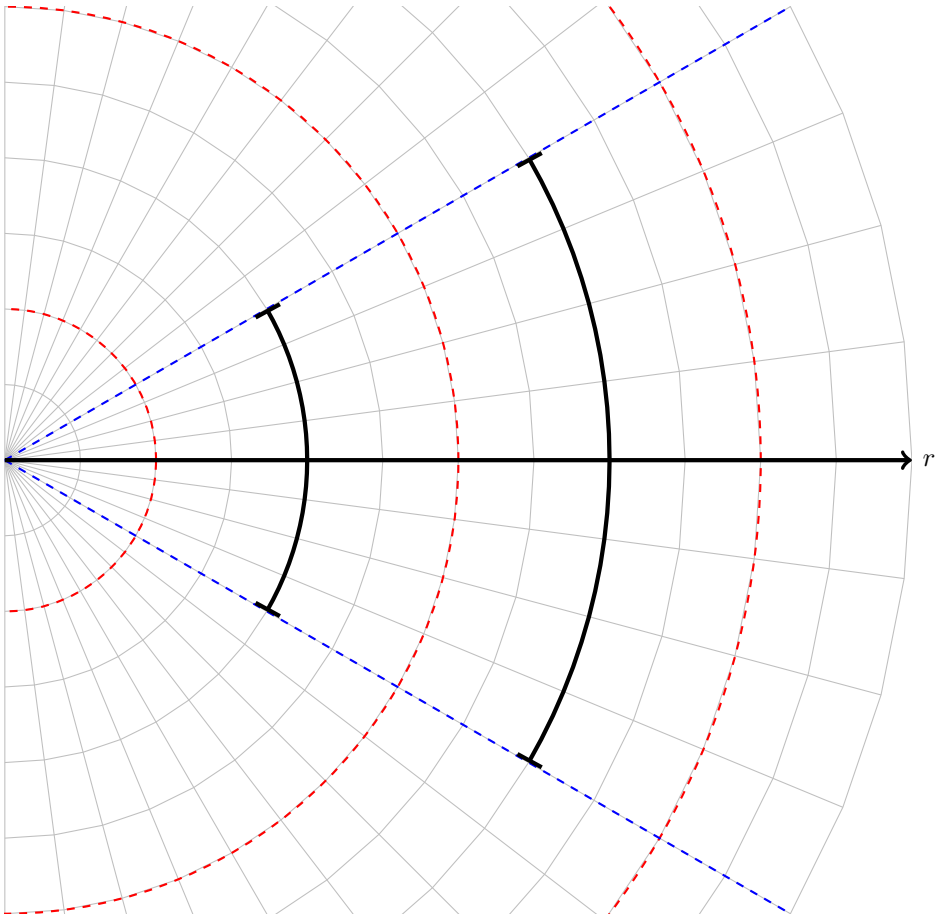

Figure 2: Curved Shims