Beam Physics Note
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Estimation of Low-Relativistic Energy Straggling in Thin Stripping Foils

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Abstract: Use of thin carbon stripping foils at TRIUMF-ISAC for charge state increase in low relativistic beams ($\beta < 10\%$) requires the consideration of energy loss and energy straggling effects due to beam-foil interaction. In this report, the estimated energy loss through the foil is computed for the MEBT case. In addition, a formula is derived, which estimates the magnitude of the energy straggling effect in the longitudinal beam energy spread due to multiple coulomb scattering inside the foil.

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In this note, an estimation of both the energy loss through the ISAC-MEBT [1] foil and the energy straggling or broadening of the beam energy spread are computed. An arbitrary beam of ^{18}O is used as an example. The RFQ output bunch distribution has been measured on-line to possess a time spread of Δt = 1.0 ns and an energy spread of $\Delta E/E$ = 0.8%. For a beam of ^{18}O with E/A = 0.153 MeV/u, this translates to (z,Pz) = (5.4 mm,8 mrad).

The stopping power for a proton at β_{RFQ} in carbon is governed by the Bethe-Bloch Equation, in this case in the low relativistic regime ($\beta = 0.0183$). For this work, the stopping power was obtained from the `PSTAR` database [2]:

$$\left\langle -\frac{dE}{dx} \right\rangle_{p^+} = 6.619 \times 10^2 [\mathrm{MeVcm^2/g}].$$

This is then scaled to the ion of interest using:

$$\left\langle -\frac{dE}{dx} \right\rangle_{I} = \frac{Z^2}{N} \left\langle -\frac{dE}{dx} \right\rangle_{p^+}. \quad (1)$$

The proton's stopping power is scaled using the atomic and neutron numbers (Z,N). The ion's stopping power is then converted to a mean expected energy loss, provided the thickness of the target material. The MEBT carbon stripping foils are quoted as having an area density of:

$$\rho_A = 5 \mu\mathrm{g/cm^2}. \quad (2)$$

The total energy loss is found by multiplying the stopping power (1) with the foil area density (2), which produces an expected foil energy loss of:

$$\Delta E_F = \left\langle -\frac{dE}{dx} \right\rangle_{I} \rho_A = 21 \mathrm{keV}.$$

Next, the magnitude of the energy straggling due to multiple coulomb scattering within the foil is estimated. This spread is due to the random collisions between ^{18}O ions and the lattice of ^{12}C atoms in the foil. Each collision results in a random energy loss ΔE_{coll}. In order to estimate the number N of total scatterings inside the foil, the interaction cross section σ for scattering between beam and foil atoms is itself estimated. The atomic radii of ^{18}O is:

$$r_o = 60pm$$

The density of carbon is ρ_C = 2.2 g/cm^3, which means that per unit volume, there are:

$$n = \rho_C / m_C \quad (3)$$

atoms in total. Here m_C is the mass of an individual carbon-12. The probability of interaction is obtained by considering a hypothetical ion in the beam whose radius is equal to $r = 2r_o$ [3]. This sphere travels through the carbon foil, whose constituents are reduced to points representing their nuclei. Over a time t, the ion will define a cylindrical volume:

$$V = \pi r^2 \beta c t = \pi (2r_o)^2 \beta c t. \tag{4}$$

The cross section of this cylinder is:

$$\sigma = \pi (2r_o)^2 \tag{5}$$

and represents the interaction cross section for scattering events between beam and foil. The mean free path between scatterings is:

$$l = \frac{1}{n\sigma}. \tag{6}$$

The thickness h of the foil can be found using ρ_A (foil area density) and ρ_C (density of ^{12}C):

$$h = \rho_A / \rho_C \tag{7}$$

which corresponds to η oxygen-carbon interaction mean free paths:

$$\eta = \frac{h}{l}. \tag{8}$$

This means we expect an average of η scatterings as beam ions transit the foil material. The average energy loss per scattering is is the total foil energy loss divided by this number:

$$\Delta E_{coll} = \frac{\Delta E_F}{\eta}. \tag{9}$$

Since the energy loss process is a random walk, the variance will be $\sqrt{\eta}$:

$$\Delta E_S = \sqrt{\eta} \Delta E_{coll} = \frac{\Delta E_F}{\sqrt{\eta}} \tag{10}$$

Putting this all together, an expression estimating the magnitude of the energy straggling effect is obtained:

$$\Delta E_S = \frac{Z^2}{N(2r_o)}\sqrt{\frac{\rho_A m_c}{\pi}}\left\langle -\frac{dE}{dx}\right\rangle_{p^+} \qquad \text{(oxygen beam, carbon foil)}$$

Where Z and N are the atomic and neutron numbers of the projectile and r_o its atomic radius, ρ_A is the foil area density, m_c the mass of an individual foil atom and the stopping power of the proton at identical E/A. This expression can be used for any low beta beam and thin stripping foil, provided the atomic parameters are adjusted.

Using this expression and the above quoted numbers, the expected broadening of the beam energy spread due to straggling is thus about 4 keV for the MEBT foil, for a foil energy loss of about 21 keV. Assuming the initial beam energy spread is $\Delta E/E$ = 0.8% with an energy of 2,754 keV (E/A = 0.153 MeV/u), post-foil beam is expected to have an energy spread of $\Delta E/E$ = (0.8% + 4/2,754) $\approx$ 0.9%, a fractional increase of about 10%. The effect of energy straggling upon the longitudinal phase space distribution is shown in Figure 1.

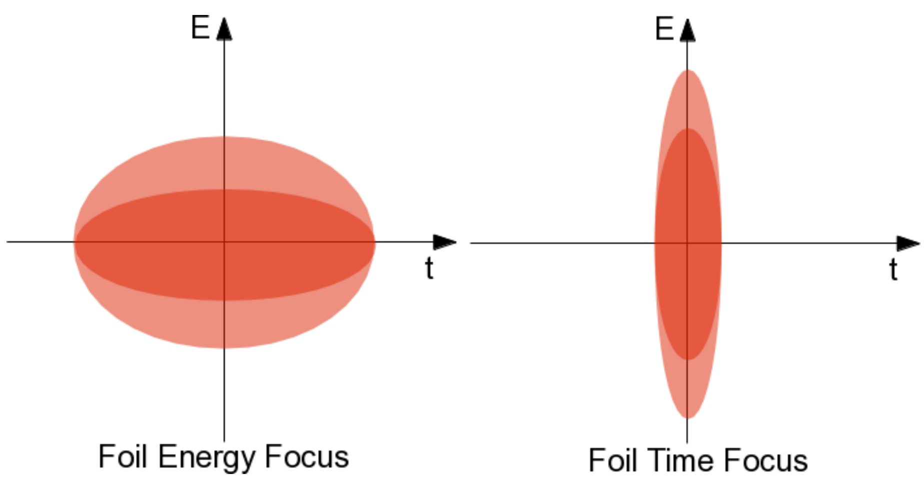

Figure 1: Representation of energy straggling on the longitudinal phase space representation of a particle beam. In this image, both an energy focus and a time focus are shown. This shows the output distribution, after foil interaction. The entire distribution has been shifted down by an energy of ΔE_F. The straggling effect (light colour) is ΔE_S.

Expected charge state distributions

To estimate the charge state distribution resulting from beam-foil interaction, we assume the average scattering energy ΔE_{coll} from Eq. (9) defines the mean of a Maxwell-Boltzmann distribution:

$$\mathcal{P}(\Delta E) = \sqrt{\frac{2}{\pi}} \frac{\Delta E^2 e^{-\Delta E^2/(2a^2)}}{a^3}. \tag{11}$$

The shape parameter a is related to the mean scattering energy via:

$$a = \frac{\Delta E_{coll}}{2} \sqrt{\frac{\pi}{2}}. \tag{12}$$

The probability P of a given integer charge state q is obtained by considering the following: during scatterings, electrons are also likely to re-combine with ions of both beam and foil. In other words, not only are electrons lost, they are also gained back. A given charge state is likely to persist only if the average scattering energy is sufficient as to prevent re-combination. Then, the probability of a given charge state is:

$$P(q = n) = \mathcal{P}\Big(\Delta E > E_n\Big) - \mathcal{P}\Big(\Delta E > E_{n+1}\Big) \tag{13}$$

and where:

$$P(\Delta E > E_n) = \int_{\Delta E_n}^{\infty} \mathcal{P}(\Delta E_i) d(\Delta E_i). \tag{14}$$

As an example, the probability of charge state 4+ is the probability that the average collisional energy is above the sum of the ionization energies for states 1 to 4, but below charge state 5.

Obtaining the ionization energies for oxygen, and using values of 5.0μg/cm^2 for the MEBT foils, the scattering energy spectrum along with the charge state probability are computed and shown in Figure 2. In the figure, the charge state probabilities computed using the detailed method are compared to a stripping computation provided by the TUDA group [4]. An ^{18}O beam of E/A = 0.153 MeV/u is used together with a ^{12}C foil. The estimated number of scatterings is 114 and the full-width energy straggling effect is $2\Delta E_S$ = 4 keV. The energy loss through the foil is ΔE_F = 21 keV. This produces an average collisional energy of ΔE_{coll} = 186 eV. The energy loss after foil transit reduces beam E/A to 0.1518 MeV/u. Figure 3 shows the dependency of ΔE_F and ΔE_S upon the foil area density, ρ_A.

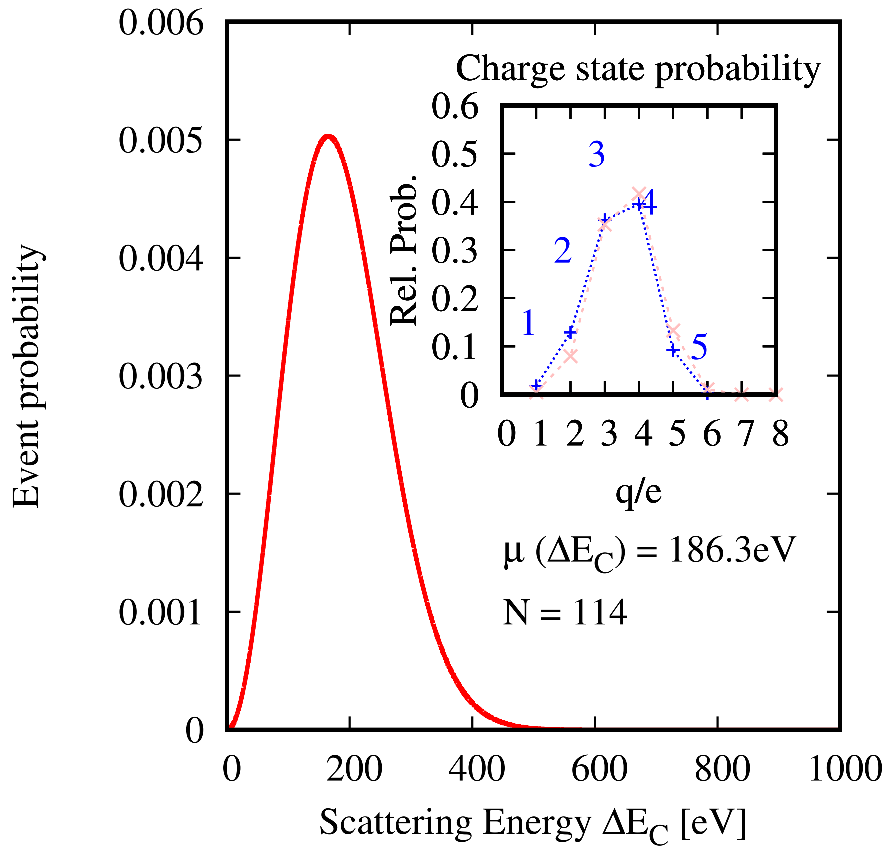

Figure 2: Computed scattering energy spectrum for an ^{18}O beam upon a 5.0μg/cm^2 ^{12}C foil, as used in the ISAC-MEBT section. The charge state probabilities are plotted in the inset graph. A comparison is made to charge state predictions provided by C. Ruiz, TRIUMF, shown in pink.

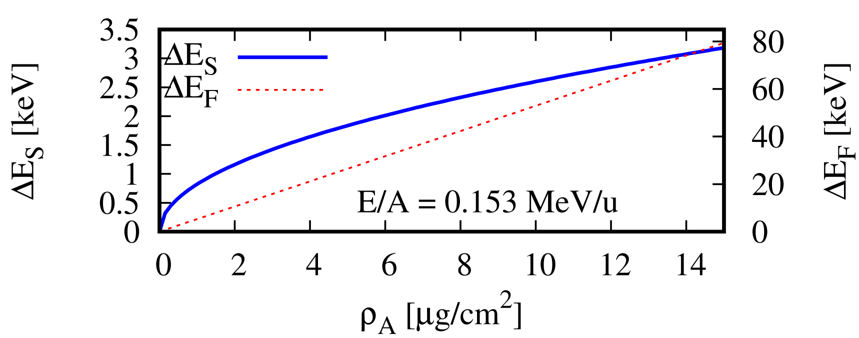

Figure 3: Magnitude of the energy straggling ΔE_S and of the foil energy loss ΔE_F versus foil area density, for an ^{18}O beam with a ^{12}C foil.

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References

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