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Magnetic Measurement of Permanent Magnet Quadrupole

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Abstract: Permanent magnet quadrupoles with integrated strength of about 0.3 T were commissioned by SABR for the DarkLight experiment at the TRIUMF e-Linac. Three of these were recently shipped to TRIUMF. Prior to their installation, it is important to have an accurate understanding of their fields, misalignments and to a lesser extent, the multipole errors. We used a Hall Probe mounted on the Beamlines Group field mapper to simulate a "rotating coil" measurement of one magnet. The subsequent data analysis is described herein. The integrated field strength was found to be about 0.33 T, approximately 10% higher than the design specifications. Alignment error was found to be of the order of 0.7 mm.

1 Field Mapping

A field mapping of the SABR permanent magnet quadrupole (PMQ) was performed in the TRIUMF magnet measurement area, with the setup shown in figure 1. The magnet that was measured was marked with the serial number 003. The radial component of the magnetic field was measured for 24 different angles (in steps fo 15 deg), for two different radii: 7.5 mm and 15 mm.

Figure 1: PMQ mounted at the magnet measurement area ready for field mapping using probe apparatus on the left.

Each individual measurement within these groups was taken at a fixed horizontal and vertical probe location and rotational angle, with the probe then moving along the longitudinal direction, recording the magnetic field strength at intervals of 5 mm. The raw measurement profiles for both groups of measurements are shown in figure 2.

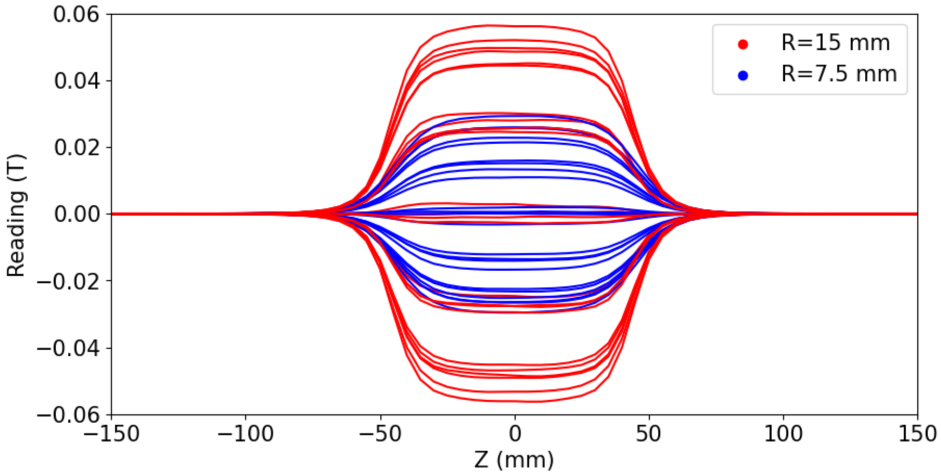

Figure 2: Raw measurement profiles obtained from magnetic field mapping.

The gradient for each measurement was extracted by taking the integral of the above magnetic profiles. This was then normalized with respect to the measurement radius and subsequently plotted as a function of probe rotation angle, which can be seen in figure 3. This plot shows the expected sinusoidal shape, with the results from both measurement groups overlapping fairly well.

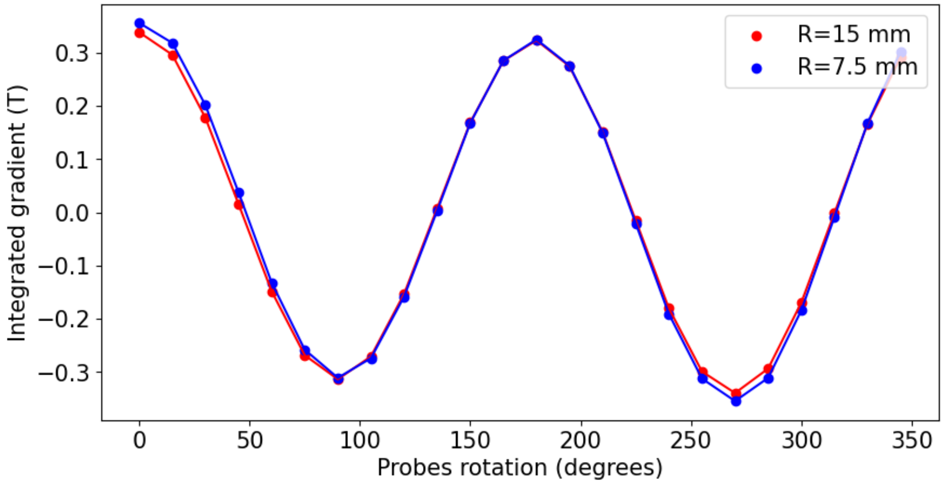

Figure 3: Integrated gradient as a function of the magnetic probe's rotation.

2 Fourier Transform

To obtain further details about the quadrupole it was necessary to take the Fast Fourier Transform (FFT) of the data in figure 3 and extract the coefficients of the expansion. This allows us to determine the amplitude and phase of the multipole field errors, in addition to the true integrated gradient and overall misalignment of the magnet. The respective integrated amplitudes of each multipole are reported in table 1: as it is often done with quadrupole magnets [1, Chapter 6], these amplitudes are scaled to the magnet full aperture $r_0 = 26\,\mathrm{mm}$, and are given in unit of the quadrupole strength.

Harmonic	**Measurement at R = 15 mm**		**Measurement at R = 7.5 mm**	
	Amplitude	**Phase (deg)**	**Apmplitude**	**Phase (deg)**
Monopole	0.002	0.0	0.001	0.0
Dipole	0.028	-62.0	0.025	-56.8
Quadrupole	1.000	0.293	1.000	-0.907
Sextupole	0.003	-109.7	0.011	-113.8
Octopole	0.004	34.6	0.041	54.3
10th pole	0.002	40.4	0.059	74.6
12th pole	0.007	84.7	0.188	162.4

Table 1: Normalized amplitudes and phase components from performing FFT on both groups of measurements.

The amplitudes are then normalized by the measurement radius and overlaid to qualify their level of agreement. This is shown in figure 4. The monopole component was omitted from this plot as it is expected to be zero. At this time we do not understand this offset: it is relatively small, it could come from a zero-offset of the hall probe; this will be further investigated but ultimately does not impact the results presented herein. For higher order multipole errors to be accurate (typically octupole and above), the measurements should be performed at a large radius with respect to the aperture of the magnet. At smaller radii, for example the measurements taken at 7.5 mm, the estimates of higher multipole elements will not be accurate, explaining the discrepancy between measurement sets at higher harmonics seen in figure 4.

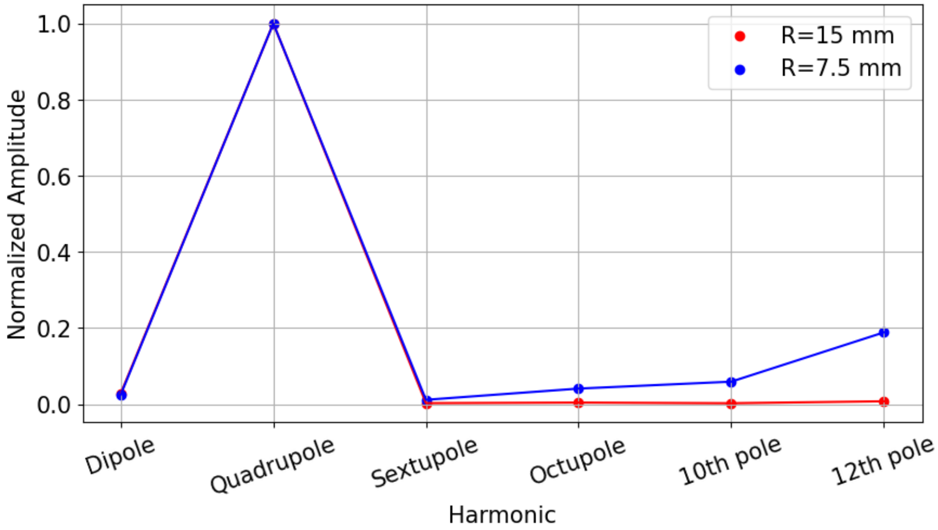

Figure 4: Normalized amplitudes of FFT components for both groups of measurements.

We extract the true integrated gradient of the PMQ by taking the amplitude of the second harmonic (quadrupole element) and dividing this by the radius of measurement. This yields the values presented in table 2. When comparing the average value to the design specifications, we note a difference of approximately 10%, which contradicts the allowance of ± 5% that SABR specified.

Radius (mm)	**Integrated Gradient (T)**
15	0.329
7.5	0.337
Average	0.333

Table 2: Extracted values of integrated gradient from FFT components.

Finally, we obtain the misalignment by taking the normalized amplitude of the first harmonic (dipole element) and multiply by the magnet aperture of 26 mm. This is reported in table 3.

Radius (mm)	**Misalignment (mm)**
15	0.719
7.5	0.637
Average	0.678

Table 3: Extracted values of magnet misalignment from FFT components.

Overall we see that the magnet has a small degree of misalignment due to a non-zero dipole harmonic, however it's dominant harmonic is clearly the quadrupole element, as is expected for a quadrupole magnet. This characterization process should be repeated for the other two magnets received from SABR so far, in addition to any subsequent magnets ordered for the DarkLight experiment.

References

[1] S. Russenschuck, Field computation for accelerator magnets: analytical and numerical methods for electromagnetic design and optimization, John Wiley & Sons, 2011.