Beam Physics Note
TRI-BN-24-16
July 10, 2024

Jacobian for the Optimization Of Dipole Gap Profiles – Second Part

Thomas Planche

TRIUMF

Abstract: The method presented in the internal note TRI-BN-23-09 was valid only for dipole magnets with non-saturated poles; In this note I extend this method to the case of a fully saturate magnets.

4004 Wesbrook Mall, Vancouver, B.C. Canada V6T 2A3 · Tel: 604 222-1047 · www.triumf.ca

1 Introduction

The premises are the same as for TRI-BN-23-09 [1] except that we now consider the case of a fully saturated magnet ($\mu_r \to 1$). More precisely, we assume that the magnetization $\vec{M}$ is uniform within the pole and oriented along y: the direction normal to the magnet median plane. This assumption allows us to represent the effect of a modification of the shape of the pole surface using a current loop. This analytical technique is described is several text books; particularly relevant to our case is it application to the design of superconducting cyclotron magnets [2].

The effect of displacing one point on the surface of a fully saturated pole can be modeled using an infinite sheet of current, running along the surface of a groove (see: [1, Fig. 1]) with a line density:

$$\frac{\mathrm{d}\vec{I}}{\mathrm{d}s} = \vec{M} \times \vec{n}\,, \tag{1}$$

where $\vec{n}$ is the a unit vector normal to the surface and pointing outward. Since $\vec{M}$ is assumed to be oriented along y and $\vec{n}$ is confined to the (x, y) plane, $\mathrm{d}\vec{I}$ is oriented along z. Basic trigonometry leads:

$$\mathrm{d}I = \pm M\frac{h}{l}\mathrm{d}s\,, \tag{2}$$

with h the height and w the half-width of the groove, and $l^2 = w^2 + h^2$. Note that if $h < 0$ the “groove” is actually a “bump”.

With $\vec{M}$ pointing upward and the groove height $h > 0$, $\mathrm{d}I$ is positive along the left side of the groove, and negative along it right side. Because we assumed that the pole is fully saturated, $M = \frac{B_\mathrm{s}}{\mu_0}$ with B_s the saturation field of the ferromagnetic material ($B_\mathrm{s} \approx 2.14\,\mathrm{T}$ for soft steel [2]).

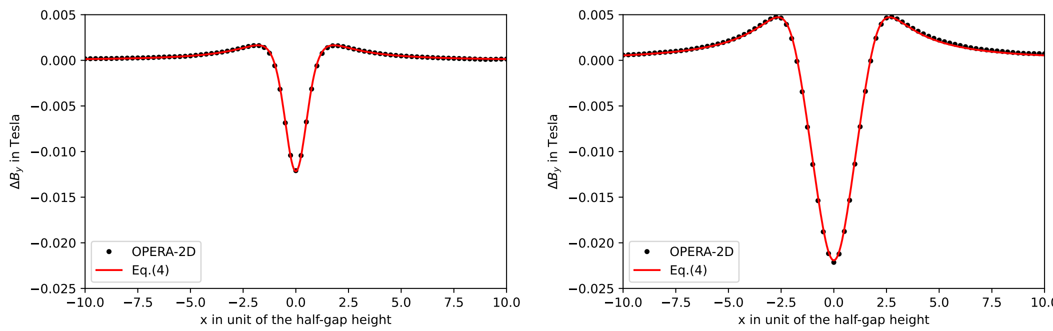
Each infinitesimal section of the groove behaves as a pair of infinitely long current-carrying wires, one below and one above the median plane. The resulting field along this plane is purely vertical and give by:

$$B_y = \frac{B_\mathrm{s} h}{\pi l}\left(\int_0^{+l} \frac{x - w\frac{s}{l}}{r^2}\,\mathrm{d}s - \int_{-l}^{0} \frac{x - w\frac{s}{l}}{r^2}\,\mathrm{d}s\right) \tag{3}$$

with $r^2 = \left(x - w\frac{s}{l}\right)^2 + \left(1 + h - h\frac{|s|}{l}\right)^2$, where all length are given in units of the half-gap height. In the limit where $h \to 0$:

$$B_y \approx h\,\left.\frac{\partial B_y}{\partial h}\right|_{h=0} = h\frac{B_\mathrm{s}}{2\pi w}\ln\left(\frac{\left(x^2+1\right)^2}{-2w^2\left(x^2-1\right) + w^4 + \left(x^2+1\right)^2}\right)\,. \tag{4}$$

To validate this model I used a simple OPERA-2D dipole model, with constant gap height. I created a groove on the surface of the pole with a depth $h = 4\%$ of the half-gap height. I set the coil current so that the pole of the magnet is fully saturated. The differential effect of the groove is shown on Fig. 1 for two different groove width, and the theoretical prediction from Eq. (4) is compared with the OPERA result.



(a) $h = 0.04, w = 0.5$ (b) $h = 0.04, w = 2.0$

Figure 1: Differential effect of a groove on the magnetic field distribution $B_y(x)$ along the median plane ($y = 0$). The position x is given in unit of the magnet half-gap height. The depth h of the groove is 4% of the dipole half-gap height. The OPERA-2D result (dots) is compared with the closed form Eq. (4) (solid lines), for two different groove width: 0.5 and 2 times the half-gap height.

2 Jacobian matrix

The Jacobian matrix coefficient for a fully saturated magnet ($\mu_r \to 1$) that we are looking for is obtained as:

$$J_1(x) = \frac{1}{B_0} \left. \frac{\partial B_y}{\partial h} \right|_{h=0}, \tag{5}$$

where B_0 is field value at ($x = 0$) with no groove. Written explicitly in units of B_0 per unit of the half-gap height it becomes:

$$\boxed{J_1(x) = \frac{B_\mathrm{s}}{2\pi B_0 w} \ln\left(\frac{\left(x^2+1\right)^2}{-2w^2\left(x^2-1\right)+w^4+\left(x^2+1\right)^2}\right).} \tag{6}$$

References

[1] T. Planche, Approximate Jacobian for the Optimization of Dipole Gap Profiles, Tech. Rep. TRI-BN-23-09, TRIUMF, `https://beamphys.triumf.ca/~tplanche/text/note/astor-magnet/conformal-mapping/report.pdf` (2023).

[2] M. Gordon, D. Johnson, Calculation of fields in a superconducting cyclotron assuming uniform magnetization of the pole tips, Part. Accel. 10 (1980) 217–222.